SURFACE DEFORMATION PRODUCED BY ION BOMBARDMENT

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Methods and results are given for the change in shape of a homogenous and isotropic body in response to ion bombardment. Possible



errors are indicated in the method of measuring [1] the erosion rate as a function of angle of incidence of the ion beam.

The rate of mass removal is defined by the sputtering factor S (atom/ion). Experimental results indicate that S varies substantially with the angle of incidence  $\varphi$  for polycrystalline materials and for energies on the order of 1 keV. Figure 1 shows a typical  $\Sigma(\varphi) = S(\varphi)/S(0)$  relation for 6 keV Ar<sup>+</sup> ions on copper. An ion beam uniform in direction, energy, and density will thus produce a change in surface shape.

An ion beam (density  $j_0$  ion/cm<sup>2</sup>-sec) moves in the negative direction of the y-axis (Fig. 2) and strikes a body whose shape is initially described by  $y_0(x)$ . The practical  $j_0$  in cathode sputtering are such that the motion of the ions is of free-molecular type, so we need not consider the difference between the actual case and the planar one. The rate of erosion along the normal at P is given for a known  $S(\varphi)$  by

$$W_n = -\frac{j_0 \cos \varphi \, \mathcal{S} \left( \varphi \right)}{N_0} = -\frac{j_0 \mathcal{S}_0}{N_0} \sum \left( \varphi \right) \cos \varphi \left( \mathcal{S}_0 = \mathcal{S} \left( 0 \right) \right),$$

in which N<sub>0</sub> (atom/cm<sup>3</sup>) is the atomic density of the material and  $\varphi$  is the angle between the normal at P and the y-axis. Then the rate of erosion along the y-axis is

$$W_y = \frac{W_n}{\cos\varphi} = -\frac{j_0 S_0}{N_0} \sum (\varphi), \qquad (1)$$

$$y(x, \tau) = y_0(x) + \int_0^{\tau} W_y(x, \tau) d\tau, \qquad (2)$$

in which  $\tau$  is time. We substitute (1) into (2) and differentiate with respect to time to get

$$\frac{\partial y}{\partial \tau} = -\frac{j_0 S_0}{N_0} \sum (\varphi).$$
(3)

It is desirable to have  $\Sigma$  as a function of  $\tan \varphi$ , i.e.,  $\partial y/\partial x$ . Differentiation with respect to x then gives

$$\frac{\partial^2 y}{\partial \tau \partial x} = -\frac{j_0 S_0}{N_0} \frac{\partial \Sigma}{\partial (\partial y / \partial x)} \frac{\partial^2 y}{\partial x^2} \,.$$

Substitution reduces this nonlinear differential equation of hyperbolic type to the form

$$\frac{\partial f}{\partial t} = -\frac{\partial \Sigma}{\partial j} \frac{\partial f}{\partial x} \qquad \left( t = \frac{\tau_{I_0} S_0}{N_0} \right), \ f = \frac{\partial g}{\partial x} \right). \tag{4}$$

We know  $\Sigma(\varphi)$ , and hence  $\Sigma(f)$ , so (4) may be put as

$$\frac{\partial f}{\partial t} = \xi(f) \frac{\partial f}{\partial x} \qquad \left(\xi(f) = -\frac{\partial \Sigma}{\partial f}\right). \tag{5}$$

The initial condition is

$$f(x, 0) = \frac{\partial y_0}{\partial x} = F(x), \qquad (6)$$

 $-F'\xi(/)$ ,

and the solution satisfying this may be put as

$$V(x,t,f) = f - F[x + t\xi(f)] = 0.$$
(7)

 $\partial V$ 

In fact

$$\frac{\partial Y}{\partial j} = 1 - F't \frac{\partial \zeta}{\partial j}, \quad \frac{\partial Y}{\partial x} = -F', \quad \frac{\partial Y}{\partial t} =$$
  
and then

$$\frac{\partial f}{\partial t} = \frac{\partial V / \partial t}{\partial V / \partial j} = \frac{-F'\xi(f)}{1 - F't \, \partial\xi / \partial j} ,$$
$$\xi(f) \frac{\partial f}{\partial x} = \xi(j) \frac{\partial V / \partial x}{\partial V / \partial j} = \frac{-F'\xi(f)}{1 - F't \, \partial\xi / \partial j} = \frac{\partial t}{\partial t} .$$

Figure 3 shows the forms of  $\Sigma(f)$  and  $\xi(f)$  for the  $\Sigma(\varphi)$  of Fig. 1. Existing data on  $S(\varphi)$  do not allow us to determine the form of  $\xi(f)$  for  $f \rightarrow 0$  with adequate precision.

If we assume that  $\Sigma(\varphi) \sim (\cos \varphi)^{-1}$  [2] for small  $\varphi$ , then  $\xi(f) \to 0$ as  $f \to 0$ . The observed  $\Sigma(f)$  is closely fitted by exp  $\{a \mid f \mid -bf^2\}$ , and in that case  $\xi(0)$  is finite but has two values. Figure 3 shows  $\Sigma(f)$  and  $\xi(f)$  for the case where  $\partial S/\partial \varphi = 0$  when  $\varphi = 0$  ( $\Sigma(\varphi) \sim (\cos \varphi)^{-1}$  for small  $\varphi$ ).

It is clear that a substantial change in  $\xi(f)$  does not produce a deviation in  $\Sigma(f)$  exceeding the experimental error.

This method has been used to determine the change in shape for some simple surfaces; the broken line for  $\xi(f)$  in Fig. 3 was used. The solution does not take account of transfer between parts of the surface, so it applies, strictly speaking to convex surfaces. Figures 4 and 5 show successive forms for bodies of initial shape  $y_0 = \cos x$  and  $y_0 = 4 \cos x$ ; it is clear that the bombardment has a leveling action.

The effects of  $\xi(f)$  for small f on the change of shape were considered via a hemisphere on the end of a cylinder,  $y_0 = (1 - x^2)^{1/2}$ , with both of the  $\xi(f)$  of Fig. 3 (Fig. 6). The results for  $\partial S/\partial \varphi = 0$  at  $\varphi = 0$  do not differ from those for  $\partial S/\partial \varphi \neq 0$  at  $\varphi = 0$  within the errors of the calculation; the surface tends to become conical with  $\lim |f| = 3.5$  for  $t \rightarrow \infty$ , which corresponds to the f at which  $\xi(f) = 0$ , i.e., to maximum erosion ( $\Sigma_{\max}$ ). Figure 7 shows the shape change from  $y_0 = \cos x$  after exposure at  $\varphi_0 = 40^\circ$  to the x-axis; ridges analogous to those observed [3] are formed.

Wehner [1] deduced  $S(\varphi)$  from the erosion of a spherical model on the assumption that the surface at a given x did not vary greatly in  $\varphi$ during bombardment, so  $\Sigma(\varphi) \sim \Delta y(\varphi)/\Delta y(0)$ . Wehner [1] did not



give the precise  $j_0$  used, so it is not possible to determine the characteristic reduced times t; but he states that  $j_0$  was on the order of 1



Fig. 3



Fig. 4













Fig. 8

mA/cm<sup>2</sup>, so we may take t as 0.2-0.4. Figure 8 compares the  $\Sigma(\varphi)$  used in the calculation on the hemisphere, curve 1, with curves 2 and 3 obtained by processing the surface forms by Wehner's method [1] for t of 0.2 and 0.4. It is clear that this processing gives an error in  $\Sigma(\varphi)$  and in the estimate of the corresponding to  $\Sigma_{max}$ .

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